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Preface to Localization and solitary waves in solid mechanics, a theme published by the Royal Society of London

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Preface

This theme issue is dedicated to the description and mathematical modelling of spatial localizations in continuum solid mechanics. Responses that localize to some portion of a distinguished coordinate (either space or time) rank perhaps next in importance to homogeneous steady states and periodicity, yet they are quite distinctive in being fundamentally and significantly nonlinear. Sharply discontinuous localization, as in brittle fracture, is also a common physical response, but we concentrate here on localizations that arise in a balance between two conflicting influences, one pinching and the other dispersive, such that they spread over a finite spatial domain — as in the formation of a plastic neck. Applications presented here range from the folding of geological strata and the buckling of cylindrical shells, twisted rods and pipelines, to the propagation of travelling solitary waves in suspended beam systems. All are describable by a new breed of mathematical theories based on the analysis of homoclinic solutions of nonlinear differential equations posed on the infinite length scale. While no material length can actually be infinite, this recognizes that if a specimen is 'long enough', the influence of the boundaries can be swamped by the homoclinic effect.

One essential ingredient of the localization described in this issue is that it occurs in inherently nonlinear systems for which there is some form of energy conservation. Such problems can often be posed in a variational formulation, leading via Euler-Lagrange equations to differential equations which are *Hamiltonian* (the analogue of a classical mechanical system described in terms of generalized coordinates and momenta). Under certain conditions, such equations may admit infinitely many *multi-modal* homoclinic orbits, effectively comprising copies of a primary 'one-humped' solution glued together at different separations. These more complex shapes can then themselves compound together in a variety of possibilities that constitute the building blocks for 'spatially chaotic' equilibrium configurations. One implication of this is that numerical methods specifically designed for localized solutions should be used to deal with the multiplicity inherent in this chaos. As well as questions of existence and computation, those of the stability of localized solutions also become of paramount importance.

We hope that this special issue will draw attention to what we believe to be significant natural phenomena that have often been masked in the past by coexisting periodic solutions. With the growth in computer power has come an increased interest in nonlinear behaviour, and consequently the need to select physically relevant solutions from competing alternatives. Much remains to be done, and this special issue should thus be seen more as a pointer to future developments than a definitive statement that might transcend time.

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